COUPLED CONVECTIVE HEAT TRANSFER FROM ANNULAR FINS IN TRANSVERSE FLOW

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One way to intensify heat transfer between smooth surfaces is to use finning. Depending on the flow conditions, a stream of internal heat transfer agent is directed along the fin, normal to its surface, or it has a nonzero angle of attack, ($0 < \gamma < 90°$). For the last two cases, as a rule, one observes separation of the outer flow and reverse flow behind the fin element. Heat transfer conditions for developed surfaces in separated flow was studied in [1-3]. We note, however, that these references considered surfaces at constant temperature. For actual heat exchangers the temperature varies over the surface and for complete modeling one must consider the coupled problem, i.e., allow for simultaneous propagation of heat both in the heat transfer agent, and in the washed surface.

In this paper, for the coupled problem we studied heat exchange conditions and determined the thermal efficiency of the fins in transverse flow ($\gamma = 90°$) for some flow regimes and finned surface geometry. We considered surfaces with annular or planar finning (Fig. 1, where a and b show inner and outer annular finning), and geometry and flow conditions such that a single vortex forms in the interfin space (Fig. 2a). These conditions were achieved at large enough Reynolds number Re and cell size, when the fin height h is comparable with the interfin distances. The hydrodynamic conditions of separated flow in cavities of this type have been studied in detail, e.g., in [4-7]. A region of potential vortex flow is formed in the cavity. This flow has a specific structure of velocity distribution, and the range of the velocity U_v in a vortex varies from zero at the vortex center to a maximum value at the periphery. In general there is a dependence $U_v = f(U_n)$ (U_n is the velocity of the external potential flow outside the cavity). According to the results of experimental and theoretical investigations [4-7], in the cavities of the geometry studied, in regions of a vortex far from the center, the relation $U_v = k_v U_n$ ($k_v = 0.3$-0.4) is valid.

We consider a cavity geometry where there is curvature in the planes of intersection of the fin and the base. This geometry is typical for specific technologies in fabrication of finned surfaces, e.g., fabrication of finned surfaces by rolling. Here at the corner points there are no stagnation zones or secondary vortex flows.

We postulate that the flow and heat transfer conditions are identical in each cell. The condition for the hydrodynamic flows to be identical in adjacent cells holds, as a rule, over an entire surface where the flow is established. An exception is the initial sections where the external flow conditions outside the interfin cells vary from cell to cell. Regarding the identical nature of heat transfer conditions, strictly speaking the postulate does not hold due to the variation of outer flow temperature from cell to cell. The temperatures in adjacent cells T_v will differ. However, as is shown below, this difference does not appreciably influence the computed results.

For the hydrodynamic flow conditions studied, as was noted by Bachelor and Squire [4], a boundary layer forms on the cell surface. The experimental investigations of the dynamic layer formed on the surface in the separated flow region, indicate a laminar boundary layer [7].

On the basis of the geometry considered and the above postulates the boundary layer is assumed to be continuous over the entire surface of the interfin cell (Fig. 2a). The boundary layer development is shown in Fig. 2b, where for simplicity of understanding the curved surface is shown as planar. The identical nomenclature of points of the wetted surface for different cells (Fig. 2) indicates identical hydrodynamic and thermal conditions in adjacent cells. The flow scheme and the formation of the boundary layer in an example of planar systems has been examined in more detail in [8].

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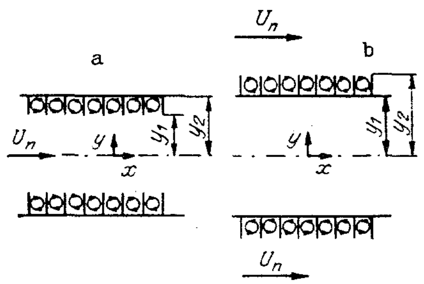

Fig. 1

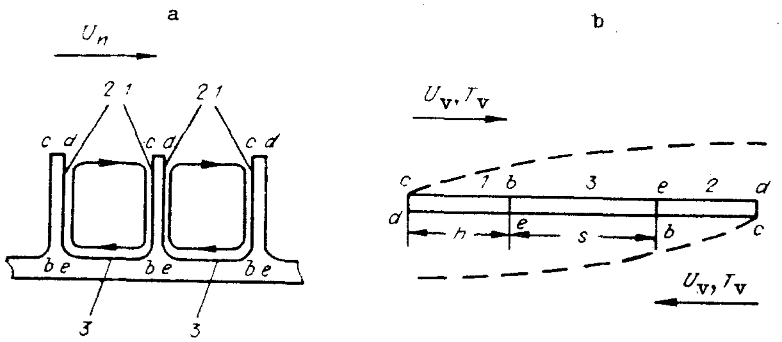

Fig. 2

Before creating a mathematical model of the heat transfer of the systems studied we note that our aim is to determine the thermal efficiency of a fin and its local thermal characteristics. The characteristics found in solving this problem enable the design of various heat exchange structures for which the fin geometry and the hydrodynamic flow conditions are the same as those studied here.

We now construct a mathematical model of heat transfer in the interfin cavity. Due to the coupled nature of the problem we must simultaneously solve the equations for transfer of mass, momentum and energy in the external heat transfer agent and of heat in the wetted object under conditions of coupling at the boundary. This approach is traditional and is used in the great majority of studies. Here we use a somewhat different approach which allows us to simplify the computing procedure to solve the coupled problem, and based on using the principle of local similarity of the boundary layer formed on the surface, and the method of superposition [9, 10]. The essence of the approach is to determine the general functional dependence between the density of heat flux removed from the surface at an arbitrary temperature, and the variation of temperature difference at the surface:

$$q_s(\Phi) = \alpha^*(\Phi)\left[T_s(\Phi = 0) - T_\mathrm{v} + \int_0^\Phi d\xi F(\Phi, \xi)\frac{dT_s}{d\xi}\right]. \tag{1}$$

As the independent variable in Eq. (1) we took the Goertler variable $\Phi = \frac{1}{\nu}\int_0^x U_n(\xi)\,d\xi$, where x is the current coordinate along the surface, (ν is the pneumatic viscosity of the heat transfer agent, ξ is the variable of integration, and $U_n(\xi)$ is the velocity in the external potential flow corresponding to the coordinate ξ. The expression for the local heat transfer coefficient at constant surface temperature $\alpha^*(\Phi)$ and the form of the influence function for a non-finned section section $F(\Phi, \xi)$ are determined by solving the boundary layer equations. According to [9, 10] we can take the influence function as $F(\Phi, \xi) = [1 - (\xi/\Phi)^{c_1}]^{-c_2}$. The coefficients c_1 and c_2 depend on the type of boundary layer, the Prandtl number Pr and the external pressure gradient. Besides Eq. (1) there is another well-known dependence, in the form of a series [10]. Dependences of the type of Eq. (1) were used to solve coupled heat transfer problems on developed surfaces in [11, 12].

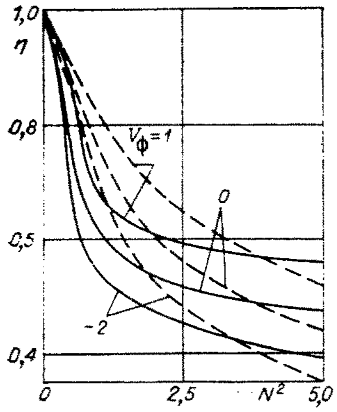

Fig. 3

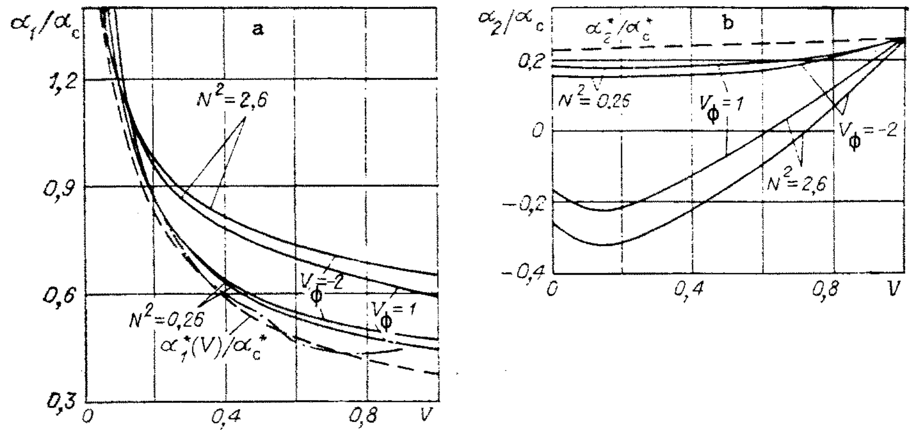

Fig. 4

In the approximation that the body is thermally thin [13] we derive an equation describing heat transfer in a circular or planar fin of variable cross section:

$$\frac{\lambda}{y^n}\frac{d}{dy}\left[\delta(y)\,y^n\,\frac{dT}{dy}\right]=q_1(y)+q_2(y). \tag{2}$$

Here y is the current coordinate; λ is the thermal conductivity of the fin material; $\delta(y)$ is its thickness; and $q_1(y)$ and $q_2(y)$ are the local heat fluxes removed from surfaces 1 and 2, respectively (see Fig. 2). For circular and planar fins n = 1 and n = 0.

Taking into account that the origin of the boundary layer is formed near the face on side 1, it is more convenient to use the relative variable y': for the external fin $y' = y_2 - y$, for the inner fin $y' = y - y_1$, and for the plane surface $y' = y$.

Assuming the base temperature to be constant and equal to T_0, we write expressions for the local heat fluxes removed from the fin surface (Fig. 2b):

$$q_1(y)=\alpha^*(y')\left[T_h-T_{\mathrm{v}}+\int\limits_0^{y'}d\xi F(y',\xi)\frac{dT}{d\xi}\right]; \tag{3}$$

$$\begin{aligned}q_2(y)=\alpha^*(2h+s-y')\Bigg[T_h-T_{\mathrm{v}}+\int\limits_{2h+s}^{h+s}d\xi F(2h+s-y',\xi)\frac{dT}{d\xi}+\\+\int\limits_h^{y'}d\xi F(2h+s-y',\,2h+s-\xi)\frac{dT}{d\xi}\Bigg]\end{aligned} \tag{4}$$

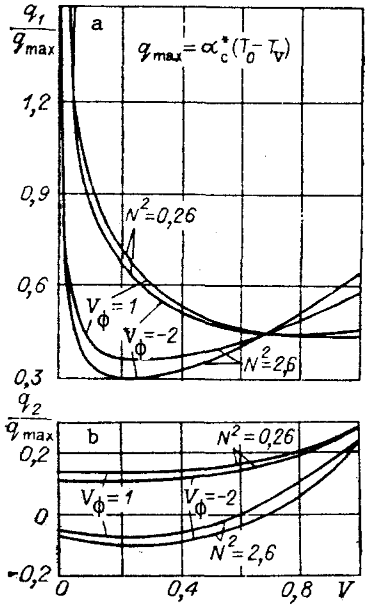

Fig. 5

(T_h is the face temperature, T_V is the temperature of the vortex core). The presence of the two integral terms in Eq. (4) reflects the fact that the thermal boundary layer on side 2 has a pre-history of development on side 1 of the adjoining fin and on the base 3 (Fig. 2), and also takes account of the absence of temperature gradient along the base.

Without accounting for the heat loss from the end surface of the fin, the boundary conditions in terms of the new variable y' have the form

$$\left.\frac{dT}{dy'}\right|_{y'=0} = 0, \quad T(y'=h) = T_0. \tag{5}$$

In dimensionless variables Eq. (2) and the boundary conditions (5), taking account of Eqs. (3) and (4), take the form:

$$\frac{1}{(V-V_\phi)^n}\frac{d}{dV}\left[\delta'(V)(V-V_\phi)^n\frac{d\theta}{dV}\right] = N^2\left\{\frac{\alpha^*(V)}{\alpha^*_c}\left[\theta_h + \int\limits_0^V d\xi F(V,\xi)\frac{d\theta}{d\xi}\right] + \right.$$
$$\left. + \frac{\alpha^*(V_0-V)}{\alpha^*_c}\left[\theta_h + \int\limits_0^1 d\xi F(V_0-V,\xi)\frac{d\theta}{d\xi} - \int\limits_V^1 d\xi F(V_0-V, V_0-\xi)\frac{d\theta}{d\xi}\right]\right\}; \tag{6}$$

$$\left.\frac{d\theta}{dV}\right|_{V=0} = 0, \quad \theta(V=1) = 1. \tag{7}$$

Here $V = y'/h$; $V_0 = 2 + s/h$; $\theta = (T - T_V)/(T_0 - T_V)$; $\delta'(V) = \delta(V)/\delta_0$; $N^2 = \alpha_c{}^*h^2/\lambda\delta_0$ is a characteristic parameter of the fin; δ_0 is the fin thickness at the base; $\alpha_c{}^* = \frac{1}{2h}\int\limits_0^h dy' \times [\alpha^*(y') + \alpha^*(2h + s - y')]$ is the average heat transfer coefficient on the surfaces of an isothermal fin; $n = 1$, $y' = y_2 - y$, $V_\phi = -y_2/h$ for external annular finning; $n = 1$, $y' = y - y_1$, $V_\phi = y_1/h$ for internal finning; $n = 0$, $y' = y$, $V_\phi = 0$ for planar finned surfaces.

In the computations as $\alpha^*(V)$ we use an expression for the heat transfer coefficient on a constant temperature surface;

$$\alpha^*(V) = g_0\frac{\lambda_T}{h}\mathrm{Re}_h^{1/2}V^{-1/2}, \tag{8}$$

where $\mathrm{Re}_h = U_V h/\nu$; λ_T and ν are the thermal conductivity and the kinematic viscosity of the heat transfer agent; and g_0 is a parameter, depending on Pr, the pressure gradient and the flow regime.

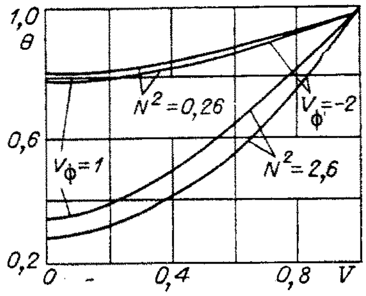

Fig. 6

The integral equation (6) with boundary conditions (7) was solved numerically by the Runge–Kutta method. The algorithm of the numerical solution for a circular fin is practically no different from the computing scheme for planar fins, as described in [8]. Taking into account that the boundary layer formed on the surface of the interfin cell is laminar, we assume $c_1 = 3/4$, $c_2 = 1/3$ [9, 10]. The computations were performed for external and internal annular fins, and also for fins on a plane surface with constant cross section $\delta'(V) = 1$. We assumed that $s/h = 1$. The results are shown in Figs. 3-6.

Before going on to analyze the computed results we should point out that the thermal characteristics found for the fins, expressed in dimensionless form, do not depend on the values of T_V, and are determined only by the parameters N^2 and V_ϕ.

One of the initial hypotheses used in constructing the mathematical model is that conditions are isothermal in adjoining cells. In particular it was postulated that the temperatures T_V are identical. However, they actually differ. The temperature in the leading cell in the flow direction T_{1V} differs from T_{2V} in the cell behind the fin. We estimate the influence of the error $\Delta T_V = T_{2V} - T_{1V}$ on the computed results. To do this we replace T_V in Eq. (3) by T_1 and T_V in Eq. (4) by T_{2V}. The form of Eq. (6) relative to $\theta = (T - T_{1V})/(T_0 - T_{1V})$ is conserved, and in the second square brackets the quantity θ_h is transformed into $\theta_h + \Delta\theta_V$, where $\Delta\theta_V = (T_{2V} - T_{1V})/(T_0 - T_{1V})$. Estimates show that over a wide range of variation of T_V and T_0 the maximum value of $\Delta\theta_V$ does not exceed $\Delta\theta_V \leq 0.1$ as a rule. As a result of the numerical calculation we found that accounting for $\Delta\theta_V$ when varying the parameters N^2 and V_ϕ leads to a computing error not exceeding 2-5%.

One of the chief integral thermal characteristics of a fin is its efficiency η, which is determined by the ratio $\eta = Q/Q_{max}$ (Q is the total heat flux removed by the fin, $Q_{max} = \alpha_c^*(T - T_V)F$ is its maximum value possible for a fin with infinitely great thermal conductivity at surface temperature T_0, and F is the fin surface area).

We shall show that with the computed $\alpha^*(V)$, according to Eq. (8), the parameter g_0 for a laminar boundary layer can be represented in the form $g_0 = A\,Pr^{0.33}$, where A depends on the pressure gradient on the wetted surface. The influence of the pressure gradient on A in cavities was not investigated, and therefore as a first approximation in the computations we used its value for zero-gradient flows.

The solid lines on Fig. 3 show the efficiency of planar and annular fins as one varies the characteristic parameter N^2, which determines the influence of the external hydrodynamic flow conditions, the geometry, and the thermophysical properties of the fin. Comparison of the curves obtained shows that in transverse flow over finned surfaces, for a given fin geometry $s/h = 1$, with other conditions equal, the highest efficiency is found for internal annular fins, and the least efficiency for fins on the outside of the tube. This conclusion is valid also for geometry $s/h \neq 1$ with the condition that a single vortex is formed in the interfin cavity.

Figure 3 also shows the efficiency of planar and annular fins as computed by the engineering methods of [13], when the heat transfer coefficient on the fin surface is considered constant and equal to α_c^* (broken lines). A comparison of the curves shows overstimated efficiencies found by the engineering methods over a considerable region of variation of the parameter N^2. An exception is the region of large values of N^2, where using α_c^* in the calculations leads to reduced values of η compared with the values obtained by solving the coupled problem.

As follows from Fig. 4, the maximum heat transfer coefficients will be in the face region of the fin on the leading side where the boundary layer begins to form. To estimate the influence of the surface temperature on the heat transfer conditions we compare the local heat transfer coefficients found by solving the coupled problem (solid lines) with their values on an isothermal surface $\alpha^*(V)$, computed from Eq. (8) (broken lines). The comparison shows that the increase of the temperature difference along the surface on the leading side of the fin leads to an increased heat transfer coefficient $\alpha_1(V)$, and that $\alpha_1(V)$ increases with increase of N^2.

Besides the computed heat transfer coefficients Fig. 4a shows the experimental values (found in [14]), expressed in the dimensionless form α_e/α_c^*, and taken for a fin geometry s/h = 1.06 (dot-dash lines). The nature of the distributions of heat transfer coefficients on the forward fin surface coincides in general with $\alpha_1^*(V)$, apart from a certain region near the fin root, this being due apparently to the different hydrodynamic conditions. The experimental model did not have the curvatures in the near-root zone that were assumed in the computed model, and in the base region of the fin secondary vortex flows or stagnation zones were formed. Further, on the surface of the base and of the trailing side of the fin the hydrodynamic conditions tested experimentally [14] and considered here differ substantially.

On the trailing side of the fin there is a further decrease of the local heat trasnfer coefficient (Fig. 4b). Typically, for $N^2 > 1.5$-2.0 one observes reversal of heat flux in regions adjacent to the face. In this region the heat transfer coefficient is negative. The reversal is due to deformation of the temperature profile in the boundary layer. This was first obtained in [15] and was examined in detail in [10, 16], and has been applied to finned surfaces for other flow conditions, e.g., in [17]. For regimes for which there is heat flux reversal the behavior of local heat transfer characteristics differs qualitatively from the analogous characteristics found by solving the uncoupled problem. Therefore, in these situations one should solve the coupled problem.

Figure 5a, b shows local heat flux distributions $q_i(V)$, $(i = 1, 2)$, removed from fin surfaces 1 and 2, respectively. As can be seen on Fig. 5a, the maximum heat flux occurs near the face on the leading side of the fin. For small N^2 $q_1(V)$ decreases monotonically from the face surface to the fin base, but as N^2 increases the values of $q_1(V)$ reach a minimum at some distance and then increase. This behavior of $q_1(V)$ and an increase of the temperature difference (Fig. 6) from the face to the base. On the trailing side of the fin $\alpha_1(V)$ is substantially less than $q_2(V)$. For large N^2 the value of $q_2(V)$ in the face region becomes negative, i.e., for example, during cooling of a finned surface in the fin sections near the face one sees heating instead of cooling. There is an unusual "pumping" of heat from the trailing fin side to the leading side, where the values of $\alpha_1(V)$ are large and heat removal is a maximum. This leads to an increase of fin efficiency η (or of the total heat flux removed by the fin) in the region of large N^2, compared with values of η computed by the simplified methods, where these effects are not accounted for.

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